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VIBRATIONS OF CIRCULAR PLATES OF RECTANGULAR ORTHOTROPY

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1. Introduction

Several studies are available on vibrating circular plates of polar orthotropy [1–4]. On the other hand very limited information is available on vibrating circular plates of rectangular orthotropy, the problem being of technical importance in several technological applications, e.g., printed circuit boards [5]. The present study deals with the determination of the fundamental frequency of transverse vibration of: (1) a solid, clamped circular plate of rectangular orthotropy, Figure 1(a); (2) a circular annular plate whose material obeys the same constitutive relations, clamped at the outer boundary and free at the inner contour, Figure 1(b).

The same polynomial co-ordinate functions which satisfy identically the outer, essential boundary conditions are used for both problems. Clearly, in the case of the doubly connected plate, the energy functional is evaluated between the inner and outer boundaries [6]. The fundamental eigenvalue is determined by means of the optimized Rayleigh–Ritz method. Good engineering accuracy is achieved in the case of an isotropic plate [7].

2. Approximate analytical solution

Using Lekhnitskii's classical notation [1] one expresses the governing functional in the form

$$J[W] = \tfrac{1}{2}\int\int\left[D_1\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + 2D_1\,\nu_2\,\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + D_2\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4D_k\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2\right]\mathrm{d}x\,\mathrm{d}y - \frac{\rho h}{2}\,\omega^2\int\int W^2\,\mathrm{d}x\,\mathrm{d}y, \tag{1}$$

where $W(x, y)$ is the amplitude of transverse vibration. As shown in previous studies [4, 8] one is able to approximate the fundamental mode of vibration of isotropic circular plates by means of the polynomial co-ordinate function

$$W \simeq W_\alpha\,(x, y) = A_0\,(a^\gamma - r^\gamma)^2, \qquad r = \sqrt{x^2 + y^2}, \tag{2a, b}$$

where γ is Rayleigh's optimization parameter.

Expression (2a) constitutes the "base function" used in many elastic stability and vibrations problems of isotropic nature [8]. The accuracy of the results can be improved by taking additional coordinate functions. One has then

$$W_\alpha\,(x, y) = \sum_{n=0}^{N} A_n\,(a^\gamma - r^\gamma) r^{\alpha n}. \tag{3}$$

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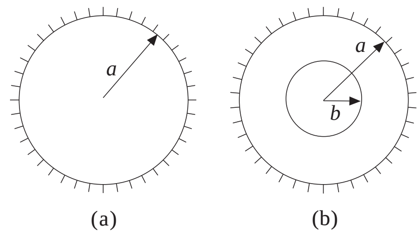

Figure 1. Vibrating structural systems under study.

In the present study numerical values of the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2$ have been obtained for $N = 0$ and $N = 1$.

Substituting equation (3) in equation (1) and requiring that

$$\partial J/\partial A_n = 0 \qquad (n = 0, 1, \ldots, N), \tag{4}$$

one obtains a homogeneous linear system of equations in the A_n's. The non-triviality condition yields a secular determinant whose lowest root constitutes the fundamental frequency coefficient. Since Ω_1 is an upper bound with respect to the exact result of the eigenvalue, by minimizing it with respect to γ one is able to optimize Ω_1.

Admittedly in the case of rectangular orthotropic plates the mode shapes are also functions of the azimuthal co-ordinate θ but, as a first order approximation, it seems reasonable to disregard this variation when determining the fundamental frequency parameter.

3. NUMERICAL RESULTS

Table 1 depicts values of Ω_1 for the isotropic plate for which

$$D_1 = D_2 = D_3 = D, \tag{5}$$

where $D_3 = D_1 \nu_2 + 2D_k$. The values of $\Omega_1 = \sqrt{\rho h/D}\omega_1 a^2$ are determined for $\nu_2 = \nu = 1/3$ in order to compare with the exact results available in reference [7]. The agreement with

TABLE 1

Values of $\Omega_1 = \sqrt{\rho h/D}\omega_1 a^2$ *in the case of an isotropic circular plate* ($\nu = 1/3$) *clamped at the outer boundary.*

	$N = 0$		$N = 1$		
b/a	γ	Ω_1	γ	Ω_1	[7]
0*	1·845	10·245	1·973	10·216	–
0·01	1·841	10·244	1·967	10·217	–
0·1	1·705	10·277	1·601	10·267	10·18
0·2	1·518	10·524	1·273	10·428	10·34
0·3	1·485	11·446	1·240	11·374	11·37
0·4	1·577	13·558	1·318	13·521	13·54
0·5	1·761	17·627	1·514	17·614	17·51
0·6	2·079	25·558	1·940	25·556	25·60
0·7	2·635	43·003	2·958	42·999	42·38
0·8	3·768	92·886	1·716	92·860	85·32
0·9	7·190	360·377	3·519	360·130	–

* Solid plate (exact value of Ω_1 is 10·2158).

TABLE 2

Values of $\Omega_1 = \sqrt{\rho h/D_1}\omega_1 a^2$ *in the case of an isotropic circular plate clamped at the outer boundary* ($D_2/D_1 = 1/2$; $D_k/D_1 = 1/3$; $\nu_2 = 1/3$).

	$N = 0$		$N = 1$	
b/a	γ	Ω_1	γ	Ω_1
0*	1·845	9·235	1·973	9·208
0·01	1·841	9·234	1·967	9·209
0·1	1·705	9·263	1·601	9·255
0·2	1·518	9·486	1·273	9·399
0·3	1·485	10·318	1·240	10·253
0·4	1·574	12·221	1·317	12·188
0·5	1·761	15·889	1·513	15·887
0·6	2·027	23·037	1·940	23·036
0·7	2·635	38·762	2·958	38·759
0·8	3·768	83·726	1·712	83·703
0·9	7·190	324·840	3·514	324·617

* Solid plate.

the exact results is very good from an engineering viewpoint except for the case where $b/a = 0{\cdot}8$, the difference being of the order of 9%. On the other hand it is difficult to assess the accuracy of the "exact" results available in the literature.

Table 2 shows values of $\Omega_1 = \sqrt{\rho h/D_1}\,\omega_1 a^2$ for orthotropic circular plates for which $D_2/D_1 = 1/2$, $D_k/D_1 = 1/3$, and $\nu_2 = 1/3$. For $b/a = 0$ (solid plate) the one term solution predicts $\Omega_1 = 9{\cdot}235$ while the two term solution yields $\Omega_1 = 9{\cdot}208$. Lekhnitskii [1, p. 432] gives an approximate expression of ω_1 which in terms of Ω_1 becomes

$$\Omega_1 = 6{\cdot}33\sqrt{1 + 0{\cdot}667 D_3/D_1 + D_2/D_1}. \tag{6}$$

This expression is, certainly, only valid for the simply connected clamped plate. For the orthotropic case under consideration equation (6) yields $\Omega_1 = 9{\cdot}317$ which is almost 1% higher than the value determined using the one term Rayleigh optimization procedure.

For a clamped solid isotropic plate, equation (6) yields $\Omega_1 = 10{\cdot}32$ while with the optimized procedure one obtains $\Omega_1 = 10{\cdot}245$, the exact result being 10·2158. The two therm optimized approach yields $\Omega_1 = 10{\cdot}216$.

The present method is quite simple and straightforward. A similar approach can be used in the case where the plate is simply supported at the outer edge.

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REFERENCES

1. S. G. LEKHNITSKII 1968 *Anisotropic Plates*. New York, N.Y.: Gordon and Breach Science Publishers.
2. P. A. A. LAURA, G. C. PARDOEN, L. E. LUISONI and D. R. AVALOS 1981 *Fibre Science and Technology* **15**, 65–77. Transverse vibrations of axisymmetric polar orthotropic circular plates elastically restrained against rotation along the edges.

3. D. J. Gunaratnam and A. P. Bhattacharya 1989 *Journal of Sound and Vibration* **137**, 383–392. Transverse vibrations and stability of polar orthotropic circular plates: high-level relationships.
4. H. A. Larrondo, D. R. Avalos and P. A. A. Laura 1993 *Journal of Sound and Vibration* **168**, 31–37. Axisymmetric vibrations and buckling of non-uniform, cylindrically aelotropic circular plates carrying a central concentrated mass.
5. D. S. Steinberg 1988 *Vibration Analysis for Electronic Equipment*. New York, N.Y.: John Wiley, second edition.
6. E. Romanelli and P. A. A. Laura 1997 *Computers and Structures* **62**, 795–797. An approximate method for analyzing transverse vibration of circular annular plates of non-uniform thickness and a free inner boundary.
7. A. W. Leissa 1969 *Vibration of Plates*. NASA SP 160.
8. P. A. A. Laura 1995 *Ocean Engineering* **22**, 235–250. Optimization of variational methods.